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Conductivity of an Impure, Nonequilibrium Plasma with Electrothermal Instabilities

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Introduction

It is generally recognized that small amounts of impurity can have a significant effect on the conductivity of a plasma.[1,2] This is particularly true for a nonequilibrium plasma, where the elevated electron temperature is determined principally by collisions between electrons and neutral atoms. A few tenths of a percent of molecular impurity can absorb a large proportion of the electron energy and lower the electron temperature.

The conductivity of a two-temperature, nonequilibrium plasma is also affected by fluctuations in the electron density, usually referred to as electrothermal instabilities. When the Hall parameter exceeds a certain critical value, oscillations in the local electron density and current density occur, reducing the effective conductivity.[3,4]

The purpose of this paper is to discuss the combined effect of impurities and fluctuations on the conductivity of a nonequilibrium plasma.

Plasma Specification

A steady-state, seeded plasma is assumed to be flowing perpendicular to a uniform magnetic field. The Hall current is assumed to be zero, so that the electric current is perpendicular to both the magnetic field and the gas velocity. Except for electron density, the properties of the plasma are uniform. The bulk gas total temperature is 2000°K, while the electron temperature is determined from a balance between Joule heating and the losses from collisions between electrons and the heavy species. For simplicity, radiation and thermal conduction are neglected. The average electron density is determined from the Saha equation. The bulk gas total pressure is 2×10^5 Newtons per square meter and the magnetic field strength is one tesla. The argon is seeded with $\frac{1}{10}\%$ cesium.

The analysis of the electron density fluctuations follows Solbes.[3] In that paper a uniform plasma consisting of a neutral carrier and a partially ionized seed is assumed to be present in a uniform magnetic field. A quasi-linear plane wave instability is analyzed and expressions for the effective conductivity of the plasma presented. The analysis in this paper uses the equations in Ref. 3 with only slight modification to include an impurity in the neutral carrier gas. The number density of the carrier is replaced by the sum of the number densities of the carrier and impurity. The electron-atom collision cross section is replaced by an average cross section based on the relative number density of carrier and impurity. The molecular weight of the carrier gas is replaced by an average molecular weight based on the mass fraction of carrier and impurity.

The only impurity discussed in this paper is carbon monoxide. It is a fairly common impurity and absorbs an appreciable amount of energy in a collision with an electron.[5] The effect of other impurities is qualitatively the same as that presented in this paper.

Discussion and Results

The electron temperature of the plasma is dependent on the current density and therefore on the electric field in the plasma. The external circuit, connected through the electrodes, controls the electric field and determines the state of the plasma. The effect of this external circuit is introduced through a dimensionless load parameter, K, defined as $K \equiv -E/UB$ where E is the electric field in the plasma, U is the gas velocity, and B is the magnetic field strength. The output power density is then given by $P_D = -JE = K(1 - K)\sigma U^2B^2$.

The variation of power density with load parameter is shown in Fig. 1. The Mach number is 0.5. Curve *a* shows the power density variation when no impurity is present and the density fluctuations are neglected. This curve represents the ideal output with nonequilibrium conductivity. The electron temperature varies inversely with the load parameter, from about 3100°K at $K = 0$ down to the gas temperature at $K = 1$.

When the instability analysis by Solbes[3] is included in the calculation, the power density variation follows curve *b*. Curves *a* and *b* are identical until the load parameter K reaches 0.2. Then the plasma becomes unstable and local oscillations reduce the conductivity. No impurity is present in curve *b*.

Curve *c* shows the reduced power density when $\frac{1}{10}\%$ impurity is added to the carrier gas. Density fluctuations are neglected and only the effects of the impurity included. The carbon monoxide impurity has a large inelastic collision cross section and energy is quickly transferred from the electrons to the impurity molecules. The result is a lower electron temperature and a drop in plasma conductivity. The addition of the impurity reduces the power density by a factor of one half.

The effect of both $\frac{1}{10}\%$ impurity and density fluctuations is shown on curve *d*. Since the generated voltage $(U \times B)$ is constant, the drop in conductivity caused by the density fluctuations reduces the Joule heating available to the electrons. This energy loss also lowers the electron temperature. The combination of these two nonlinear effects, the impurity and the density fluctuations, lowers the electron temperature more than the sum of the two effects acting singly. The maximum value of power density is about one eighth that of curve *a* and about one fifth that of the impurity alone (curve *c*). For comparison the power density at equilibrium con-

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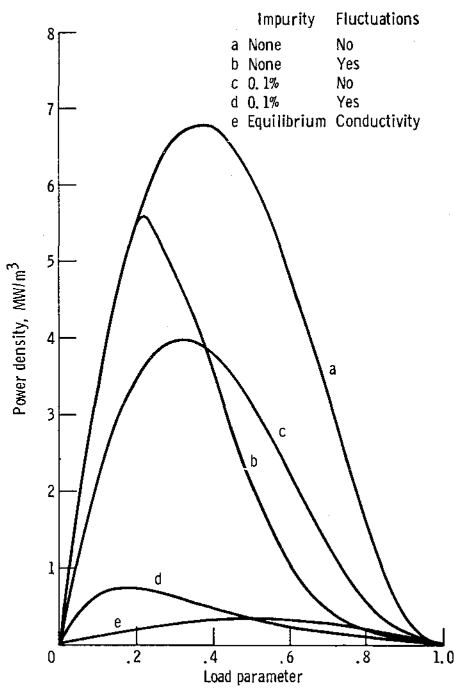

Fig. 1 The power density as a function of load factor with impurity and electron density fluctuations as parameters. Working fluid argon seeded with cesium; total pressure, 2×10^5 Newtons per square meter; total temperature, 2000°K; Mach number, 0.5; magnetic field, 1.0 tesla.

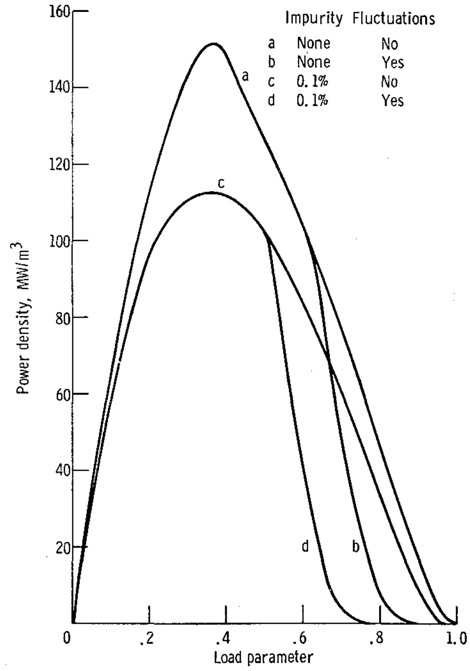

Fig. 3 The power density as a function of load parameter with impurity and electron density fluctuations as parameters. Working fluid, argon seeded with cesium; total pressure, 2×10^5 Newtons per square meter; total temperature 2000°K; Mach number, 2.0; magnetic field, 1.0 tesla.

ductivity (electron temperature equal to bulk gas temperature) is shown as curve *e*.

Figure 1 demonstrates the dramatic effect of impurities on the conductivity of a nonequilibrium plasma. The reduction in conductivity is particularly severe when the instability losses are included. The magnitude of this reduction is shown in Fig. 2, where the plasma conductivity is a function of impurity particle fraction. A load parameter of 0.25 is used for the Mach 0.5 curve. The solid curve indicates what the conductivity would be if the oscillations were absent. The dashed curve is the conductivity predicted by an analysis similar to that of Solbes[3] which includes the effect of electron density fluctuations.

Note that an impurity particle fraction of 2×10^{-4} reduces the conductivity by 10% when fluctuations are neglected, but that the reduction is 40% when fluctuations are included. This means that the impurity level becomes much more critical when instabilities are considered in the analysis. As the impurity level drops below 10^{-5} the conductivity reaches a constant value of about nine tenths of its ideal value.

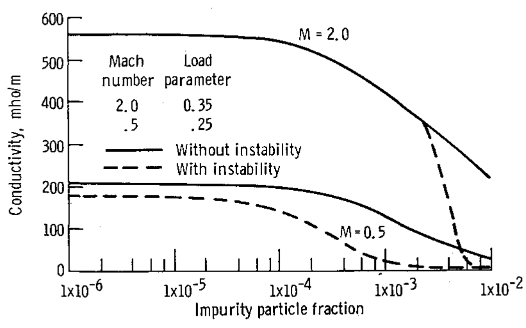

Fig. 2 The conductivity of the plasma as a function of impurity particle fraction. Working fluid argon seeded with cesium; total pressure, 2×10^5 Newtons per square meter; total temperature, 2000°K; magnetic field, 1.0 tesla.

Figure 3 shows the results of a calculation similar to that of Fig. 1, but with an increased Mach number of 2.0. The curves show an improvement in power density, particularly at low values of the load parameter K. According to the analysis by Solbes[3] the critical Hall parameter increases with electron temperature. At the higher Mach number the electron temperature and critical Hall parameter are larger, suppressing the instability over a wider range of K values.

However, the authors have some reservations about the validity of these results. At this higher Mach number the degree of ionization of the seed is much higher than in Fig. 1. For $K < 0.4$ the seed is fully ionized, and for $K < 0.7$ the degree of ionization is greater than 30%. This violates one of the assumptions in Solbes' analysis, which assumes that the degree of ionization is small. Solbes makes this assumption in order to linearize the contribution of electron temperature variations to electron density fluctuations. In fact, this linearized term becomes very large as the degree of ionization approaches one. This implies that large fluctuations in electron temperature, as well as density, may be present when a large proportion of the seed is ionized.

It is possible that a different mode of instability occurs at this high degree of ionization. Such an instability has been proposed to explain rotating spoke phenomena in MPD arc thrusters.[6] This mode involves large fluctuations in electron temperature, but relatively small fluctuations in electron density. This mode is also mentioned briefly by Haines and Nelson,[7] and may explain the results of Velikhov et al.[8] The results presented in Fig. 3 are valid if it is assumed that this temperature fluctuation mode is absent.

A plot of conductivity vs impurity fraction for a Mach number of 2.0 at $K = 0.35$ is given in Fig. 2. The comments above concerning degree of ionization and temperature fluctuation mode apply to these curves as well.

This analysis was developed for use with the closed-cycle MHD generator at Lewis Research Center. To the authors knowledge there are no experimental results with which to compare this analysis.

Conclusions

An analysis of the effect of impurities on the conductivity of a two-temperature, nonequilibrium plasma indicates that 1) the impurity reduces the conductivity of the plasma, particularly if electron density fluctuations are present, 2) at low Mach number ($\sim$0.5) the impurity particle fraction must be less than 2×10^{-5} in order to get the maximum conductivity, 3) increasing the Mach number improves the characteristics of the plasma if the temperature fluctuation mode is absent. The conductivity increases and the instabilities are more easily suppressed, 4) high current densities provide the best power density characteristics in the plasma, because of the higher electron temperature they induce.

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